Should Banks Create Money?*

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Abstract

How costly are proposals such as 100% reserve narrow banking or a system with a central bank digital currency, which constrain private banks' ability to issue deposits in excess of their reserve holdings? The paper shows that, in contrast to widespread concerns, if banks have economically viable financing alternatives to deposits, constraining bank deposit issuance must not increase loan rates and thus crowd out bank lending and intermediation. Instead it can be costly due to lower interest payments on deposits, which provide less protection for deposit-holders against inflation. A calibration to the US economy suggests however, that these costs are relatively small, in particular if bank reserves are remunerated.

Keywords: *Fractional reserve banking, money supply, narrow banking, CBDC*

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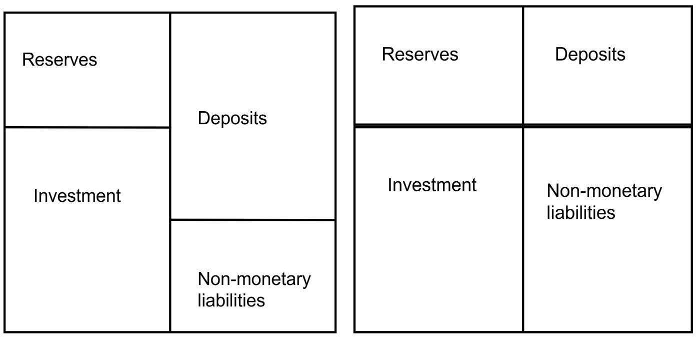
1 Introduction

Current monetary systems are characterized by a mixture of public and private provision of means of payment. Households and firms primarily pay using digital liabilities of private financial intermediaries, mainly demandable bank deposits. Banks themselves settle these transactions with digital reserves issued by the central bank, to which non-banks do not have access. Typically banks "create money", i.e. they issue deposits in excess of their reserve holdings and operate under fractional reserve banking. This traditional division of labor has been increasingly debated in recent years. For instance, the Swiss "Vollgeld" referendum in June 2018 aimed to prohibit private money creation by requiring banks to fully back their deposits with reserves, i.e. to introduce a 100% reserve banking system. Less drastic central bank digital currency (CBDC) proposals want the central bank to offer digital money to non-banks as well. This indirectly constrains the issuance of deposits as banks face competition from a CBDC. Both proposals belong to a broader debate on narrow banking (NB), i.e. the idea that financial intermediaries fully backing their demandable debt with very safe and liquid assets should play a greater role in the financial system.[1]

Figure 1: Fractional and 100% reserve banking



(a) Fractional reserve banking (b) 100% reserve banking

How costly is it to constrain fractional reserve banking (FB)? To assess this question, the paper mainly compares FB to the benchmark case of a 100% reserve narrow banking (NB) system, as illustrated in figure 1, using a general equilibrium model building on Lagos and Wright (2005). The analysis abstracts from financial stability issues.[2] In the model, households acquire money

[1]Pennacchi (2012) provides a good overview on NB and Monnet et al. (2022) on CBDC proposals. Currently 81 central banks are exploring a CBDC and 73 are running experiments or pilots (Iorio et al., 2024). The central bank might issue CBDC indirectly over specially regulated narrow banks as in Williamson (2022), or directly as official digital fiat currency. How "narrow" the financial system becomes with a CBDC, depends on how much non-banks substitute deposits for CBDC. In the extreme, i.e. if they completely switch from deposits to CBDC, a system with a CBDC is equivalent to a full NB system like 100% reserves.

[2]That FB creates the possibility of self-fulfilling bank runs and financial instability, has been well understood and extensively studied since the seminal paper by Diamond and Dybvig (1983). By eliminating the maturity mismatch from deposit-financed investment, 100% reserve NB would prevent these runs at least for the payment system. The benefits of FB (or the costs of constraining it) are less well understood. While Brunnermeier and Niepelt (2019) or

to buy consumption goods, but idiosyncratic liquidity shocks divide them into savers, who don't need money, and borrowers, who need more. Banks, who either operate in a fractional or a 100% reserve system, intermediate between these needs and diversify risk from liquidity shocks. They lend to borrowers, issue liquid deposits and illiquid bonds. While deposits circulate as means of payment, bonds, which can be interpreted as longer-term saving deposits or wholesale funding, play the role of non-monetary liabilities in figure 1. Note that a switch from FB, where deposits also finance investment or lending, towards 100% reserve banking implies that banks must substitute deposit-financed investment with funding from non-monetary liabilities, which are typically more expensive. This illustrates the widespread concern, e.g. studied by Keister and Sanches (2022), Chiu et al. (2023) or Williamson (2022) in the CBDC context, that constraining FB leads to higher funding costs and to bank disintermediation, i.e. a crowding out of private investment and lending. Chiu et al. (2023) highlighted the role of banks' market power in determining the effect on bank intermediation. The model thus also includes bank market power in the deposit market similar to Chiu et al. (2023). Finally, a central bank issues currency to households and potentially interest-bearing reserves to banks, controls inflation and the nominal interest rate.[3]

The first main insight of the model is to show that, if banks have economically viable financing alternatives to deposits, constraining FB must not lead to bank disintermediation through higher loan rates and lower bank lending. Despite the shift from deposit-financed lending to more expensive bond financed lending highlighted in figure 1, introducing 100% reserve banking does not increase the loan rate. The same holds if the central bank issues an interest-bearing CBDC. Crucial for this result is that banks' financing alternatives to deposits, which are bonds in the model, have an economically useful role. Here, bonds provide insurance against the liquidity shocks. After a liquidity shock, savers have idle reserve holdings they don't need. For them, illiquid bonds with a higher return than deposits are a useful savings vehicle and since the bond rate equals the nominal interest rate in equilibrium, savers who acquire bonds are perfectly compensated for the money holding costs. On the margin it is thus bond-holders and not deposit-holders who determine the loan rate and the switch from deposit to bond funding does not affect the loan rate.[4] The model thus rationalizes the idea that introducing 100% reserve NB or a CBDC might not significantly

Pennacchi (2012) argue FB has no benefits that could not be obtained within a carefully designed NB system, others like Ceccetti and Schoenholtz (2014) believe it is essential to banking and increases investment and intermediation. The fact that FB dominates monetary history also suggests a socially useful function. However, Chari and Phelan (2014) show how an inefficient FB system can persist over time due to pecuniary externalities.

[3] In section 4.3 I also consider a version where households have access to an interest-bearing CBDC.

[4] This is in contrast Chiu et al. (2023) or Keister and Sanches (2022) where higher deposit rates can increase the loan rate and lead to disintermediation. However, in Chiu et al. (2023) financing lending over bonds – although it is possible – is not used in equilibrium and in Keister and Sanches (2022) banks cannot issue other liabilities than deposits. Furthermore, a decrease in lending or disintermediation must not mean a decrease in welfare. In fact, it can be welfare increasing if there was over-investment initially, a point also made by Williamson (2022)

affect loan rates because financial intermediaries can finance investment with other, less liquid liabilities than deposits. This confirms similar results from partial equilibrium banking models like Whited et al. (2022) from a general equilibrium perspective. It also complements the analysis by Chiu et al. (2023), which shows that if banks have market power in the deposit market, constraining FB (in the form an interest-bearing CBDC) must not lead to disintermediation. Here, constraining FB must not lead to disintermediation if banks have viable financing alternatives to deposits, independent of bank market power.

The second insight is that constraining FB can still be costly because it leads to lower interest payments on deposits. The mechanism is as follows: as highlighted above, under FB deposits also fund loans while under NB deposits only fund reserves. Since loans have a higher return than reserves in equilibrium, this implies that banks generate higher income from deposit-financed assets under FB. Under sufficient competition, they pass on this higher income to their deposit-holders in the form of higher interest payments. This is economically useful because it better compensates the deposit-holders against inflation. A calibration to the US economy estimates these welfare costs from lower interest on deposits under NB (measured as how much consumption or GDP households would give up in a FB economy to go to the welfare level of a 100% reserve NB economy) at 0.03% of GDP for a nominal interest rate of 5%, and at 0.29% (0.61%) of GDP for nominal interest rates of 10% (15%) if bank reserves are unremunerated. With remunerated reserves, the sizeable numbers for high nominal interest rates drastically decrease. For instance, at a nominal interest rate of 15%, welfare costs decrease from 0.61% of GDP to 0.02%-0.03% of GDP, depending on how closely reserve remuneration follows the nominal interest rate. Reserve remuneration is important, because under NB, where deposits only fund reserves, banks can only pay interest on deposits with remunerated reserves.

In sum, the paper suggests that the costs of constraining FB – and thus the costs of NB proposals like 100% reserves – are relatively small. To the extent that NB regulations mitigate the inherent fragility of FB, (footnote 2 suggests this is reasonable at least for the payment system) this suggests that achieving greater financial stability with a NB system might not be so costly.[5]

The rest of the paper is organized as follows: Section 2 introduces the basic environment. Section 3 then presents the model with perfect and imperfect competition. Section 4 calibrates the imperfect competition model to the US economy and quantifies the welfare costs of 100% reserve NB and the effects of an interest-bearing CBDC. Section 5 concludes.

[5]Current FB systems exhibit various regulations to mitigate the financial instability of FB, most notably deposit insurance. Ultimately, the costs and benefits of NB regulations should thus be compared to the costs and benefits of deposit insurance and other financial stability regulations. Jackson and Pennacchi (2021) provide interesting steps in this direction.

2 Environment

The environment builds on Lagos and Wright (2005) and in particular on Berentsen et al. (2007). Time is discrete and continues forever. There are three types of agents: a continuum of households with measure 1, a continuum of private banks with measure 1,[6] and a central bank. Every period is divided into two sequential competitive markets called first market (FM) and second market (SM).[7] In each market there is a perishable consumption good, q in the FM and x in the SM.

Households are infinitely lived and discount future periods with β. At the beginning of every period they face a preference shock which divides them into two groups. With probability $s \in (0, 1)$ a household becomes a seller in the FM and produces with linear disutility $c(q_s) = q_s$. With the inverse probability a household is a buyer in the FM and consumes with utility $u(q_b)$, where $u'(0) = \infty$, $u'(q_b) > 0$ and $u''(q_b) < 0$. Let buyers' relative risk aversion $\sigma = -\frac{q_b u''(q_b)}{u'(q_b)} < 1$ be constant and below one.[8] In the SM all households consume x and produce h with quasi-linear utility $U(x) - h$, where $U'(0) = \infty$, $U'(x) > 0$ and $U''(x) < 0$. Households lack commitment and cannot enforce debt repayments. As a result, buyers cannot issue debt to buy goods from sellers in the FM and households find it useful to hold a means of payment or money.

Before describing the money supply by the central bank and commercial banks, we quickly derive the first-best allocation. With perishable consumption goods a social planner chooses non-negative consumption and production to maximize the expected period welfare of a household subject to feasibility,

$$\max_{q_b, x_b, h_b, q_s, x_s, h_s} \quad (1 - s)\left[u(q_b) + U(x_b) - h_b\right] + s\left[-c(q_s) + U(x_s) - h_s\right]$$

$$s.t. \quad (1 - s)q_b = sq_s \tag{1}$$

$$(1 - s)x_b + sx_s = (1 - s)h_b + sh_s \tag{2}$$

where (1) and (2) equalize aggregate consumption to aggregate production in the FM and the SM. The efficient quantities q_b^*, q_s^*, and $x_b^* = x_s^* = x^*$, then equalize marginal utility of consumption and marginal disutility of production in both markets:

[6] In section 3.5 with imperfect competition there will be a finite number N of banks.

[7] Since both markets are competitive I follow Berentsen et al. (2007) and call these markets first and second market instead of the more common notations decentralized market (DM) and centralized market (CM) in the New Monetarist literature.

[8] This ensures real deposit holdings increase in the deposit rate so banks face a tradeoff when issuing deposits under imperfect competition. The calibration yields $\sigma = 0.19$, so the data supports this assumption.

$$\frac{u'(q_b^*)}{c'(\frac{1-s}{s}q_b^*)} = 1, \quad q_s^* = \frac{1-s}{s}q_b^*, \quad U'(x^*) = 1. \tag{3}$$

The central bank issues fiat money M or reserves. Reserves are potentially interest bearing and grow at rate $\gamma = M/M_{-1}$.[9] To reflect the fact that only reserves held by commercial banks are interest-bearing, while non-banks hold central bank money in the form of unremunerated currency, I assume reserves are remunerated only for banks at rate $i_r \geq 0$, while for households they are unremunerated.[10] The central bank manages the supply of reserves by lump-sum transfers τ to households in the SM. Thus the central bank seignorage revenue in period t, $M - M_{-1}$, which is negative if the central bank reduces the money supply, is used for transfers τ and interest payments $i_r R_{-1}$ where R are the reserve holdings of banks.

Let $1/\phi$ be the price of x in terms of reserves in the SM and let p be the price of q in terms of reserves in the following SM. Also define inflation ϕ_{-1}/ϕ. Under stationarity the real value of money is constant, $\phi M = \phi_{-1}M_{-1}$, and the money growth rate γ equals the inflation rate. The central bank thus perfectly controls long-run inflation and the nominal interest rate, defined as $1 + i = \gamma/\beta$, the product of inflation γ and the real interest rate $1/\beta$. Throughout the paper I assume holding reserves is costly, i.e. the opportunity costs for holding money (i) are higher than the return of reserves for households (0) and the return of reserves for banks (i_r):

$$i > 0 \quad , \quad i > i_r. \tag{4}$$

Commercial banks are risk-neutral firms, owned by households. Banks can commit and monitor households at no cost. This enables them to make loans and issue debt. Specifically, banks offer loans at interest rate i_l and they issue deposits, subject to a reserve requirement $\alpha \in (0, 1]$, and bonds at interest rates i_d and i_b. All financial contracts are nominal, formed after the preference shock in the FM, and fully redeemed in the following SM.[11] Deposits are the other means of payment besides reserves, while bonds are illiquid.[12] As a benchmark the paper first considers competitive banking, before moving to imperfect competition in section 3.5, with a fixed number

[9] For brevity I denote variables in a representative period t without subscript, the period before t with subscript -1 and the period after with $+1$.

[10] To explore the implications of an interest-bearing CBDC I introduce remunerated reserves for households in section 4.3. The ECB has remunerated bank reserves since its establishment, while the FED started to pay interest on reserves in October 2008 only.

[11] With quasi-linear utility in the SM there is no gain from spreading the redemption of debt or the repayment of loans over multiple periods. Thus assuming this kind of contracts is not constraining in this environment.

[12] Deposits and bonds might also be interpreted as two more or less liquid forms of deposits, like checking and time deposits.

N of banks and Cournot competition in the deposit market following Chiu et al. (2023).

3 The Model

3.1 Banks

Banks acquire reserves r and loans l by issuing deposits d and bonds b. They maximize their profits π, subject to the reserve constraint and a balance sheet constraint:

$$\max_{\{l,r,d,b\}\geq 0} \pi = r(1+i_r) + l(1+i_l) - d(1+i_d) - b(1+i_b) \tag{5}$$

$$s.t. \quad \alpha d \leq r \quad , \quad l + r = d + b$$

We focus on the case with positive loans and conjecture in equilibrium $i_l \geq i_d$ and $i_l > i_r$, i.e. financing loans with deposits is profitable and loans have a higher return than reserves. Then the reserve constraint binds and we can rewrite the objective function as:

$$\max_{d,b} \; d\left(\alpha i_r + (1-\alpha)i_l - i_d\right) + b\left(i_l - i_b\right).$$

Any equilibrium with positive demand for deposits and bonds from households thus involves

$$i_d = \alpha i_r + (1-\alpha)i_l \tag{6}$$

$$i_b = i_l \tag{7}$$

and banks making zero profits. Note that the deposit rate is a weighted average between i_r and i_l , and that under FB $i_d \in (i_r, i_l)$ while under NB $i_d = i_r$, since deposits are only backed by reserves. Also note that if $i_l > i_r$ (which holds in equilibrium), we get the following relations between the interest rates, consistent with the assumptions above,

$$i_l = i_b > i_d \geq i_r \tag{8}$$

and bonds are more expensive than reserves.

Since the reserve requirement binds, this also determines the monetary system. If $\alpha < 1$ banks issue more deposits than reserves and we have FB and if $\alpha = 1$ banks fully back deposits with reserves and we have a NB system.

3.2 Buyers

Let $V_b(m)$ denote the value of a household entering period t with m reserves, who just learned he is a buyer. Also let $V(m_{b+1})$ denote the expected value of a buyer entering period $t+1$ with m_{b+1} reserves. The problem of a buyer reads:

$$
\begin{aligned}
V_b(m) = & \max_{\{q_b, x_b, h_b, m_{b+1}, l_b, d_b, m_b, b_b\} \geq 0} \quad u(q_b) - h_b + U(x_b) + \beta V(m_{b+1}) \\
s.t. \quad & m + l_b \leq b_b + m_b + d_b && (9) \\
& pq_b \leq m_b + d_b(1 + i_d) && (10) \\
& h_b + \phi\left(m_b + d_b(1 + i_d) - pq_b\right) + \phi(\tau + \pi) + b_b\phi(1 + i_b) && (11) \\
& = x_b + \phi m_{b+1} + l_b\phi(1 + i_l)
\end{aligned}
$$

After the preference shock buyers first acquire financial assets and borrow from banks. They use their reserves m and borrow l_b to acquire bonds b_b, reserves m_b and deposits d_b as shown in (9). In the FM buyers either use reserves or deposits to buy goods from sellers, see (10).[13] When buyers enter the SM they get resources from working h_b, from monetary wealth left over after the FM, $m_b + d_b(1 + i_d) - pq_b$, from transfers of the central bank and profits from banks, $\phi(\tau + \pi)$, and from their bond holdings $b_b\phi(1 + i_b)$. They use these resources to acquire consumption goods x_b, new money holdings ϕm_{b+1} and repay loans $l_b\phi(1 + i_l)$ as shown in (11).

Let δ denote the Lagrange multiplier for (9) and λ the Lagrange multiplier for (10). Substituting (11) into the objective function, the problem yields the following first-order conditions, assuming interior solutions for q_b, x_b, m_{b+1} and l_b:

[13]Note that p is the price of q in terms of reserves in the *next* SM. This is why the interest rate i_d shows up in the budget constraint. One might think of $m_b + d_b(1 + i_d)$ as the "monetary wealth" of a buyer in the following SM.

$$
\begin{aligned}
&q_b: && u'(q_b) = p\,(\phi + \lambda) && (12)\\
&x_b: && U'(x_b) = 1 && (13)\\
&m_{b+1}: && \beta V'(m_{b+1}) = \phi && (14)\\
&l_b: && \delta = \phi(1 + i_l) && (15)\\
&d_b: && (1 + i_d)(\phi + \lambda) \leq \delta && (16)\\
&m_b: && \phi + \lambda \leq \delta && (17)\\
&b_b: && \phi(1 + i_b) \leq \delta && (18)
\end{aligned}
$$

To consume in the FM, buyers want to hold either reserves or deposits. Since deposits are interest bearing and reserves are not (for households), the marginal benefit of deposits, the left-hand side of (16), is higher than the marginal benefit of reserves, the left-hand side of (17). Thus buyers prefer holding deposits, (16) binds and (17) is slack. Combining (16) and (15) the multiplier of the liquidity constraint reads:

$$
\lambda = \phi\left(\frac{1 + i_l}{1 + i_d} - 1\right)
$$

Thus as long as $i_l > i_d$ (which we know is true from the bank problem), buyers use all their deposits in the FM and they are liquidity constrained. We also know the bond rate equals the loan rate. This means buyers could increase their borrowing and acquire bonds with these additional funds. This does not affect the real allocation so I assume buyers do not hold bonds. Finally from (13) buyers consume the efficient quantity in the SM, and from (14) the expected marginal value of new reserve holdings equals marginal costs. Summarizing, the solution to the buyer problem reads:

$$
u'(q_b) = p\phi\frac{1 + i_l}{1 + i_d} \tag{19}
$$

$$
x_b = x^* \tag{20}
$$

$$
\beta V'(m_{b+1}) = \phi \tag{21}
$$

$$
d_b = \frac{pq_b}{1 + i_d} \tag{22}
$$

$$
l_b = d_b - m \tag{23}
$$

$$
m_b = 0 \quad , \quad b_b = 0 \tag{24}
$$

This also yields the envelope condition to the buyer problem:

$$V_b'(m) = \delta = \phi(1 + i_l) \tag{25}$$

3.3 Sellers

The problem for sellers is analogue to the buyers' problem, except for the liquidity constraint because sellers don't consume in the FM:

$$\begin{aligned} V_s(m) = \max_{\{q_s, x_s, h_s, m_{s+1}, l_s, d_s, m_s, b_s\} \geq 0} \quad & -c(q_s) - h_s + U(x_s) + \beta V(m_{s+1}) \\ s.t. \quad & m + l_s \leq b_s + m_s + d_s \\ & h_s + \phi\left(m_s + d_s(1 + i_d) + pq_s\right) + \phi(\tau + \pi) + b_s\phi(1 + i_b) \\ & = x_s + \phi m_{s+1} + l_s\phi(1 + i_l) \end{aligned}$$

Again I replace h_s in the objective function with the budget constraint.[14] Assuming interior solutions for q_s, x_s and m_{s+1} we get the following first-order conditions:

$$\begin{aligned} & q_s: && p\phi = c'(q_s) \\ & x_s: && U'(x_s) = 1 && (26) \\ & m_{s+1}: && \beta V'(m_{s+1}) = \phi && (27) \\ & l_s: && \delta \leq \phi(1 + i) \\ & d_s: && (1 + i_d)\phi \leq \delta \\ & m_s: && \phi \leq \delta \\ & b_s: && \phi(1 + i_b) \leq \delta && (28) \end{aligned}$$

Since sellers don't need liquidity in the FM, their choice of financial assets is only driven by return considerations. From the banking problem and (8) we know $i_b > i_d$ and $i_b > i_r$. Therefore sellers prefer holding bonds over deposits and reserves, (28) holds with equality and $d_b = m_b = 0$. Because $i_b = i_l$, sellers are indifferent to borrow more funds and acquire additional bonds. As with buyers, I exclude this type of borrowing. From (26) also sellers consume the efficient amount in the SM,

[14] This assumes $h_s > 0$ which requires a scaling condition on $U(.)$. As we will see sellers also consume the efficient quantity in the SM $x_s = x^*$. So $h_s > 0$ holds if x^* is sufficiently big.

and the first-order condition for new reserve holdings, (27), is identical to the condition for buyers, (14). In particular, the condition does not depend on the household's type or their wealth in the SM. So buyers and sellers choose the same reserve holdings in the SM: $m_{b+1} = m_{s+1} = m_{+1}$. This is an implication of quasi-linear utility introduced by Lagos and Wright (2005). We can summarize the solution to the seller problem as:

$$c'(q_s) = p\phi \tag{29}$$

$$x_s = x^* \tag{30}$$

$$\beta V'(m_{+1}) = \phi \tag{31}$$

$$d_s = l_s = m_s = 0 \tag{32}$$

$$b_s = m. \tag{33}$$

The envelope condition is:

$$V_s'(m) = \delta = \phi(1 + i_b) \tag{34}$$

3.4 Equilibrium

We focus on stationary equilibria where real allocations are constant over time and first derive households' optimal reserve holdings in the SM and all market clearing conditions. Iterating (34) and the envelope condition of buyers (25) one period forward, the optimality condition for households' reserve holdings in the SM rewrites as:

$$\begin{aligned} \phi = \beta V'(m_{+1}) &= \beta s V_s'(m_{+1}) + \beta(1-s)V_b'(m_{+1}) \\ &= \beta s \phi_{+1}(1 + i_{b+1}) + \beta(1-s)\phi_{+1}(1 + i_{l+1}) \end{aligned} \tag{35}$$

The marginal cost of reserves, ϕ, equals the marginal expected benefits for sellers and buyers. For sellers the marginal benefit is the real bond rate because they deposit reserves to acquire bonds. For buyers the marginal benefit is the real loan rate because if they bring more reserves they can borrow less and they save on the loan interest payments. Market clearing for reserves in the SM then reads:

$$(1-s)m_{b+1} + s m_{s+1} = m_{+1} = M \tag{36}$$

Suppose banks in total supply D deposits, B bonds, and hold L loans and R reserves. Market clearing for reserves, bonds, deposits and loans in the banking period then reads:

$$R + (1-s)m_b + sm_s = R = M_{-1} \tag{37}$$

$$(1-s)b_b + sb_s = sM_{-1} = B \tag{38}$$

$$(1-s)d_b = D = \frac{R}{\alpha} = \frac{M_{-1}}{\alpha} \tag{39}$$

$$(1-s)l_b = L = D + B - R = \left(\frac{1-\alpha}{\alpha} + s\right) M_{-1} \tag{40}$$

From (37), the reserve holdings by banks, R, and the reserve holdings of households after interacting with the banks, must equal the total amount of reserves brought into the period by households, M_{-1}. Since $m_b = m_s = 0$, banks hold all reserves after the banking period. In the bond market (38) only sellers hold bonds, which must equal total supply B from banks. In the deposit market (39) only buyers hold deposits. The total supply must equal R/α due to the binding reserve requirement and since banks hold all the reserves from (37) this is M_{-1}/α. In the loan market (40) demand from buyers equals $D + B - R$ by the balance sheet of the banking sector which using the expressions for D, B and R is also a function of total reserves. Ultimately, the FM and the SM clear according to (1) and (2).

To solve for equilibrium interest rates we now combine the expression for optimal reserve holdings, (35), with the relations for interest rates from the bank problem, (6) and (7), and apply stationarity where $\gamma = \phi/\phi_{+1}$. This yields:

$$i_l = i_b = i \tag{41}$$

$$i_d = (1-\alpha)i + \alpha i_r \tag{42}$$

The equilibrium loan rate and the bond rate equal the nominal interest rate, and as conjectured in the banking problem $i_l > i_r$ and $i_l > i_d$ hold. The deposit rate decreases in the reserve requirement, thus it is higher under FB, where $\alpha \in (0,1)$ and deposits also finance loans, than under NB, where $\alpha = 1$ and deposits only finance reserves.

To get equilibrium FM consumption combine optimal consumption (19) with optimal production (29) and use the equilibrium expressions for the interest rates and market clearing (1):

$$\frac{u'(q_b)}{c'(\frac{1-s}{s}q_b)} = \frac{1+i}{1+i_d} \tag{43}$$

The rest of the equilibrium allocation is given by:

$$q_s = \frac{1-s}{s} q_b$$

$$\phi D = (1-s) q_b \frac{c'(q_s)}{1+i_d} \quad , \quad \phi M_{-1} = \alpha \phi D \quad , \quad \phi L = \left(\frac{1-\alpha}{\alpha} + s \right) \alpha \phi D \tag{44}$$

$$x_b = x_s = x^*$$

$$h_s = x^* + \phi M_{-1} - \frac{\phi D(1+i_d)}{s} - \phi M_{-1}(1+i) \tag{45}$$

$$h_b = x^* + \phi M_{-1} + \frac{\phi L(1+i)}{1-s} \tag{46}$$

From (45) sellers work in the SM to consume and acquire real balances and get resources from redeeming deposits and bonds. From (46) buyers work in the SM to consume and repay their loans. Since SM consumption is always efficient at x^*, we can focus on the FM allocation (43) for welfare analysis. Note that FM consumption decreases in the spread between i and i_d and without spread, the allocation achieves the efficient allocation (3).[15] Therefore the welfare comparison between FB and NB depends on which system provides a higher deposit rate. Proposition 1 summarizes the main welfare implications:

Proposition 1. *Suppose holding money is costly, the banking system is fully competitive and either operates under FB with $\alpha \in (0,1)$ or under NB with $\alpha = 1$. Then there is a stationary equilibrium in which:*

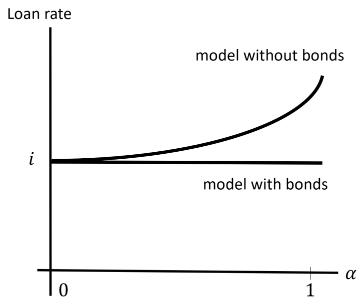
i) The loan rate $i_l = i$ is independent of α (no disintermediation).

ii) The deposit rate i_d and welfare decrease in α, thus $W_{FB} > W_{NB}$ (welfare costs of NB).

The first insight from proposition 1 is that constraining FB must not lead to disintermediation through higher loan rates which crowd out lending and investment. As proposition 1 and figure 2 show, the loan rate is the same under FB and NB despite banks substitute deposit-financed lending with bond-financed lending going from FB to NB. Figure 2 also shows, the reason lies in the availability of financing alternatives to deposits (bonds). Without bonds, moving towards NB (increasing α) also increases the loan rate, thus we have disintermediation. Appendix A.2 sketches the equilibrium in an economy without bonds.

[15]From the strict concavity of $u(q)$, the left-hand side of (43) decreases in q_b, so a higher spread must imply a lower q_b.

Figure 2: Disintermediation with and without bonds



Bonds are economically useful here because they provide insurance against the preference shocks. After the preference shock, sellers have idle reserve holdings they don't need in the FM. For them, illiquid bonds with a higher return then deposits are a useful savings vehicle. In fact, since in equilibrium $i_b = i$, sellers who acquire bonds, are perfectly compensated for the money holding costs i. Thus bonds provide perfect insurance in both banking systems and q_b in (43) is independent of s.[16] Since bonds are essential, it is the bond-holders and not the deposit-holders who determine the loan rate on the margin, as we see from (35), and the shift to bond-financed lending does not affect the loan rate. The second insight from proposition 1 is that constraining FB – although it must not lead to disintermediation – can still be costly due to lower interest payments on deposits. The reasoning is as follows: Since $i_l = i > i_r$, i.e. the return on loans is higher than the return on reserves, and under FB deposits also fund loans while under NB deposits only finance reserves, banks have a higher income on their asset side under FB than under NB. With perfect competition they will pass on this higher income to the deposit holders in form of higher i_d. This is beneficial because it compensates the deposit holders against inflation or the costs of holding money.

3.5 Imperfect Competition

Following Chiu et al. (2023), I assume Cournot competition with N banks in the deposit market, while the bond and loan market are still perfectly competitive. This means an individual bank j anticipates the effects of her own deposit supply on the deposit rate over market clearing, taking the deposit issuance of other banks $i \neq j$ as given, but ignores similar effects on the loan and the bond rate. From the perspective of bank j the total real supply of deposits is given as

$$\phi D = \phi \left(\sum_{i \neq j}^{N} d_i + d_j \right) \tag{47}$$

[16] Appendix A.1 shows, the NB allocation is identical to an economy without preference shock and banks.

where d_j is the own supply of deposits and $\sum_{i\neq j}^{N} d_i$ is the supply of all other banks. The bank anticipates total real demand for deposits from buyers from (19), (22) and (29) as

$$\phi D = \begin{cases} (1-s)\frac{q_b u'(q_b)}{1+i} & \text{if} \quad i_d \geq 0 \\ 0 & \text{if} \quad i_d < 0. \end{cases} \tag{48}$$

Buyers want to hold deposits only for non-negative deposit rates. Otherwise they switch from holding deposits to holding reserves and the demand for deposits is zero.

Equalizing (47) and (48) yields a connection between the deposits issued by bank j and the deposit rate, which I denote as $i_d(\phi D)$, where ϕD is given by (47). The interesting case is when the real demand for deposits (48) increases with the deposit rate, i.e. when $\partial\phi D/\partial i_d > 0$ and $\partial i_d(\phi D)/\partial\phi D > 0$. This means if the bank issues more deposits, buyers must be compensated with a higher return for holding these additional deposits. Only in this case the bank faces a trade-off when issuing deposits.[17] From (48) the real demand for deposits increases in the deposit rate if σ, the coefficient of relative risk aversion, is below 1, which we assumed in the environment.[18]

The problem of representative bank j is very similar to the problem under perfect competition (5). Again for $i_l \geq i_d$ and $i_l > i_r$ the reserve constraint binds and the banking problem reads

$$\max_{d_j, b_j} \quad d_j\left((\alpha i_r + (1-\alpha)i_l - i_d(\phi D)\right) + b_j\left(i_l - i_b\right),$$

where the first order condition for deposits becomes

$$(1-\alpha)i_l + \alpha i_r - i_d = i_d^{PC} - i_d = \phi d_j \frac{\partial i_d(\phi D)}{\partial \phi D}, \tag{49}$$

and i_d^{PC} denotes the deposit rate under perfect competition. The bank now takes into account that the deposit rate depends on how many deposits she issues, and i_d lies below i_d^{PC}.[19]

In a symmetric equilibrium all banks issue the same amount of deposits, i.e. $d_j = d_i = d = D/N$. (49) then becomes the familiar equality between a type of Lerner index and the inverse interest-elasticity of real deposits multiplied by the number of banks N (Freixas and Rochet, 2008, p. 78–80).

[17] In the opposite case, i.e. if a bank issues more deposits and buyers are willing to hold these deposits at a lower deposit rate, the bank would always issue as many deposits as she could, such that i_d is as low as possible, i.e. $i_d = 0$.

[18] The calibration yields $\sigma = 0.19$. So the data supports this assumption.

[19] The spread implies banks make positive profits in equilibrium, distributed to households in the SM. This does not affect the equilibrium allocation as long as sellers still work in the SM, see also footnote 14.

$$\frac{i_d^{PC} - i_d}{i_d} = \frac{1}{\varepsilon_D(i_d)} \frac{1}{N} \quad \text{where} \quad \varepsilon_D(i_d) = \frac{i_d}{\phi D} \frac{\partial \phi D}{\partial i_d} \tag{50}$$

From (50) the spread $i_d^{PC} - i_d$ decreases with competition and the elasticity of deposit demand. Note that it goes to zero if the banking sector becomes perfectly competitive, i.e. if $N \to \infty$, or if deposit demand is perfectly elastic, i.e. if $\varepsilon_D \to \infty$. Denoting the solution of (50) with i_d^*, i_d solves:

$$i_d = \max\{i_d^*, 0\} \leq i_d^{PC}. \tag{51}$$

(51) reflects the characteristics of deposit demand in (48). If competition or the elasticity are very low, banks want to set $i_d^* < 0$. But then buyers prefer to hold reserves and banks loose the cheap funding over deposits. Therefore they set the deposit rate to the lowest possible value such that buyers are indifferent, which is zero since reserves for households are unremunerated. Proposition 2 summarizes the welfare implications of the two banking systems under imperfect competition:

Proposition 2. *Suppose holding money is costly, there is Cournot competition with N banks in the deposit market and the banking system either operates under FB with $\alpha \in (0, 1)$ or under NB with $\alpha = 1$. Then there is a stationary equilibrium in which:*

i) $W_{FB} > W_{NB}$ and $i_{d,FB} = i^*_{d,FB} > i_{d,NB}$ if banking is sufficiently competitive, i.e. if $N > \bar{N}_{FB}$ where

$$\bar{N}_{FB} = \frac{\phi D}{(\partial \phi D / \partial i_d)} \frac{1}{i_d^{PC}} \tag{52}$$

is the competition level N solving (50) at $i_d^* = 0$ with $\alpha \in (0, 1)$.

ii) $W_{FB} = W_{NB}$ and $i_{d,FB} = i_{d,NB} = 0$ if $N \leq \bar{N}_{FB}$.

Proposition 2 refines proposition 1. With imperfect competition FB does not always dominate NB in terms of welfare. Although banks still generate higher income on the asset side under FB, they only pass on this higher income to deposit holders if competition is sufficiently high. In this case the imperfect competition deposit rate under FB, $i^*_{d,FB}$ is strictly higher than the deposit rate under NB, which either lies at the unconstrained imperfect competition rate $i^*_{d,NB}$ or at zero. If competition is below the threshold $\bar{N}_{FB}$, banks would like to set the deposit rate below zero under FB and are thus constrained by the outside option of households to hold unremunerated reserves.

This must hold also under NB and thus in this case banks set the deposit rate to zero in both monetary systems and the welfare is the same.

4 Quantitative implications

This section calibrates the imperfect competition model to the US economy to quantify the welfare costs of 100% reserve NB and to assess the implications of an interest-bearing CBDC.

4.1 Calibration

Following the literature I parametrize $u(q) = q^{1-\sigma}/(1-\sigma)$, $c(q) = q$ and $U(x) = B\log(x)$. This implies the stationary allocation is given by

$$x^* = B \tag{53}$$

$$q_b = \left(\frac{1+i_d}{1+i}\right)^{1/\sigma} \tag{54}$$

$$\phi D = (1-s)\frac{(1+i_d)^{\frac{1-\sigma}{\sigma}}}{(1+i)^{\frac{1}{\sigma}}} \tag{55}$$

$$i_d = \max\left\{0, \frac{1+i_d^{PC}}{1+\frac{\sigma}{1-\sigma}\frac{1}{N}} - 1\right\} \tag{56}$$

$$z = \frac{D}{D + x^*/\phi} = \frac{1}{1+\frac{B}{\phi D}} \tag{57}$$

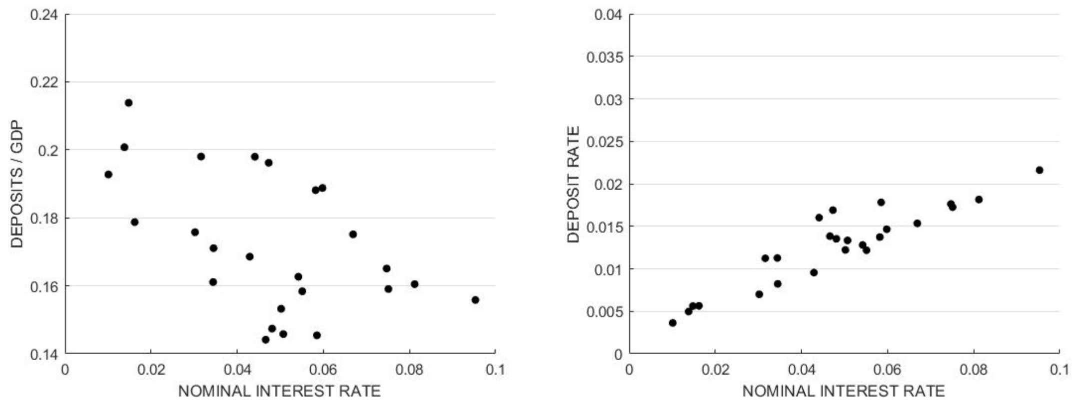
where z is the ratio of deposits to nominal GDP, i.e. aggregate real money demand or inverse velocity.[20]

Figure 3 plots US money demand and the deposit rate from 1984 to 2008 against the 3-month T-Bill rate as a measure of the nominal interest rate i. In the following I interpret every combination of real money demand and i, and the deposit rate and i as a temporary steady state of the model, generated in particular by exogenous changes in i by the central bank.[21] Thus (57) and (56) are the model equivalents thought to generate the data from figure 3. Note that money demand decreases in i, which is consistent with the model since $\partial\phi D/\partial i < 0$, and the deposit rate increases in i, but much less than one-to-one due to the imperfect transmission of the policy rate to deposit rates. To account for this spread it is crucial to use the imperfect competition model from section 3.5.

[20] Total money balances held are total deposits D which also equal nominal output in the FM. Nominal output in the SM is given by x^*/ϕ so nominal GDP is given by $D + x^*/\phi$.

[21] For similar approaches in the New Monetarist literature see in particular Lagos and Wright (2005), Craig and Rocheteau (2008) and Berentsen et al. (2015). Lucas (2000), Lucas (2013) and Lucas and Nicolini (2015) make similar arguments with a cash-in-advance model.

Figure 3: Money demand and deposit rates US 1984–2008



The calibration uses yearly, averaged US data from 1984 to 2008.[22] Before the 1980s the Glass-Steagall Act prohibited banks to pay interest on deposits and only in 1982 they were allowed to issue interest-bearing money market deposit accounts (MMDA). Following Lucas and Nicolini (2015) I assume it took the banks two years to adjust to this new type of account and from 1984 the model mechanism was operative. I stop after 2008 because nominal interest rates were close to zero afterwards.

Table 1: Measurement of model variables

Model	**Data**
deposits D	M1 − currency + MMDA
reserves M	M0 − currency
nominal interest rate i	3-Month T-Bill rate
reserve rate i_r	interest on reserves
deposit rate i_d	weighted interest of deposit components

Data sources: FRED, Lucas and Nicolini (2015).

Table 1 shows how the model variables are measured. Deposits include MMDA because they perform a similar economic function and money demand is not stable without this correction (Lucas and Nicolini, 2015). Reserves are M0 minus currency. Since banks essentially held required reserves αD in the considered period, I measure the reserve ratio as $\alpha = M/D$. The nominal interest rate is the 3-month T-bill rate and since the FED didn't pay interest on reserves until October 2008 I directly set $i_r = 0$. The deposit rate is calculated based on data from Lucas and Nicolini (2015) on the average deposit rates of M1 deposits and MMDA accounts, weighted by the shares of the two aggregates.[23]

[22] Using HP-filtered data with with $\lambda = 100$ yields practically identical parameter estimates.
[23] For the few years where there is no interest rate data for both series, I use a linear extrapolation.

With i, i_r and α taken directly from the data, the parameters left to calibrate are s, σ, B and N. They are chosen to minimize the sum of squared residuals between money demand and the deposit rate from the model given by (53) to (57) and the data counterparts z_t and i_{d_t}:[24]

$$\min_{s,\sigma,B,N} \quad SSR = \sum_{1984}^{2008} (z - z_t)^2 + \sum_{1984}^{2008} (i_d - i_{d_t})^2 \tag{58}$$

To take into account possibly changing degrees of competition over the sample period of 25 years I further split the data in two subperiods, with a degree of competition N_1 for the first and N_2 for the second subperiod. Table 2 shows the results of the calibration and figure 4 shows the model fit. It turns out that splitting the data between 1984–1990 and 1991–2008 provides the best fit. It also turns out that (58) only identifies a constant $k = B/(1-s)$ in the denominator of z, which different combinations of s and B can satisfy.[25]

Table 2: Calibrated parameters

$B/(1-s)$	σ	N_1 1984–1990	N_2 1991–2008	SSR
3.92	0.19	5	6	10.1432

The parameters from table 2 are broadly in line with similar calibrations.[26] Note that $N_2 > N_1$ indicates increasing competition in the sample period. This seems plausible given the deregulation of the financial sector at that time. I take the parameters from table 2 with $N_2 = 6$ and $\alpha = 0.04$ (average reserve ratio from 1984–2008) as the baseline calibration.

[24]Calibrating the model to match data moments or minimizing the sum of squared relative residuals yields very similar results.

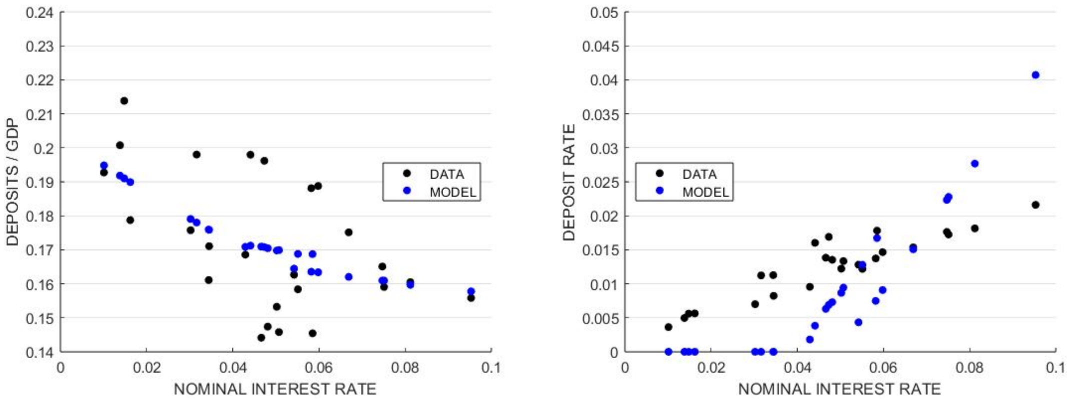
[25]As explained below, k is sufficient to calculate welfare effects and we don't need explicit values for s and B.

[26]Lagos and Wright (2005) find $\{\sigma, k\} = \{0.16, 3.94\}$, Craig and Rocheteau (2008) find $\{\sigma, k\} = \{0.14, 3.64\}$, and Berentsen et al. (2015) find $\{\sigma, k\} = \{0.31, 4.48\}$ for similar specifications. Chiu et al. (2023) find $\{\sigma, k, N\} = \{1.31, 3.73, 26\}$.

Figure 4: Model fit

(a) Money demand

(b) Deposit rate



4.2 Welfare costs of narrow banking

I first assess the welfare costs of a 100% reserve NB system. To measure them, (59) calculates the fraction Δ_{FB} of expected steady state consumption or GDP households would give up under FB to be at the welfare level of a NB economy with $q_{b_{NB}}$ given by (54) with $\alpha = 1$. The welfare costs of NB are then $1 - \Delta_{FB}$.[27]

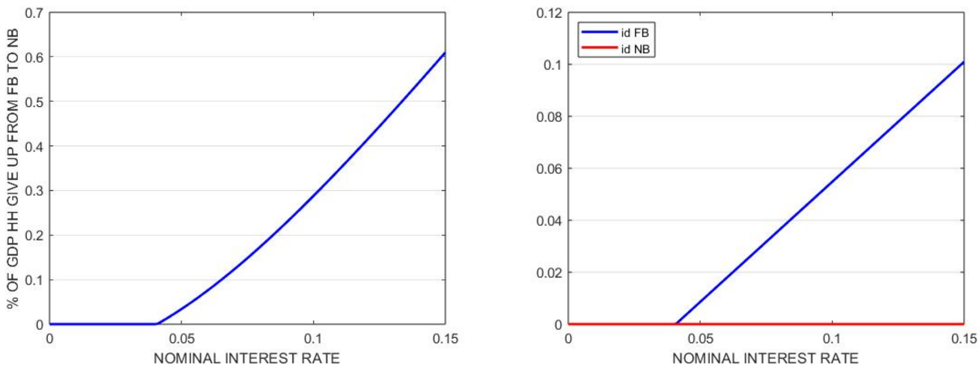
$$\begin{aligned} W_{FB}(\Delta_{FB}) &= (1-s)\frac{(q_b \cdot \Delta_{FB})^{1-\sigma}}{1-\sigma} - sq_s + B(\ln(B \cdot \Delta_{FB}) - 1) \\ &= (1-s)\frac{(q_{b_{NB}})^{1-\sigma}}{1-\sigma} - sq_{s_{NB}} + B(\ln(B) - 1) = W_{NB} \end{aligned} \tag{59}$$

We first look at the case, where the central bank pays no interest on reserves, i.e. $i_r = 0$. Think of a monetary system like in the US before 2008 where the FED was not allowed to remunerate bank reserves. Figure 5 shows the welfare costs and deposit rates under FB and NB as a function of the nominal interest rate i.

[27] Note that the single values of B and s do not matter for this welfare calculation, only the constant $k = B/(1-s)$ does. This is due to the natural log specification of SM utility which implies $\ln(B \cdot \Delta_{FB}) - \ln(B) = \ln(\Delta_{FB})$.

Figure 5: Wefare costs of NB: unremunerated reserves

(a) Welfare costs (b) Deposit rates



Notes: Baseline calibration with $i_r = 0$.

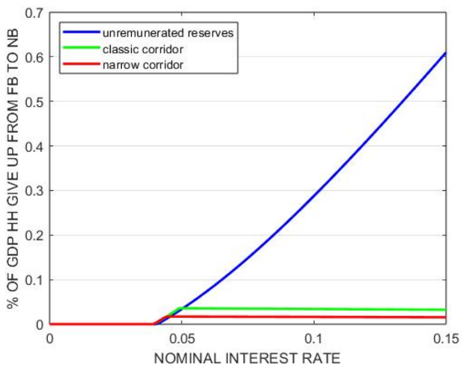
Figure 5a shows the welfare costs of NB increase with the nominal interest rate, but they are only sizeable for high inflation rates. At a nominal interest rate of 4.8% (US average 1984–2008) the welfare costs are around 0.03% of GDP, but they go up to 0.29% (0.61%) of GDP for $i = 10\%$ ($i = 15\%$). As explained above these welfare results depend on the ability of NB and FB to generate interest on deposits, which are shown in figure 5b. Under FB i_d is zero for low i and increases linearly in i from about $i = 4\%$. This is the non-monotonicity from imperfect competition we encountered in section 3.5 and which is shown in equation (56). For the baseline calibration if $i < 4\%$ banks would like to set $i_d^* < 0$, but then buyers would switch from holding deposits to holding unremunerated reserves and the demand for deposits would be zero. To avoid this, banks set the deposit rate to zero. For $i > 4\%$ banks set $i_d = i_d^* > 0$, which increases in i. Note that i_d^* is always higher than the deposit rate under NB, where banks are always constrained by the zero-lower-bound of 0 and cannot pay any interest with unremunerated reserves.

Now we turn to the case when the central bank remunerates bank reserves. Suppose the central bank operates in a corridor system and remunerates reserves at a fixed spread s below the nominal interest rate, i.e. $i_r = i - s$. We consider two versions: a classic corridor, where $s = 100$ bps, and a narrow corridor where $s = 50$ bps.[28]

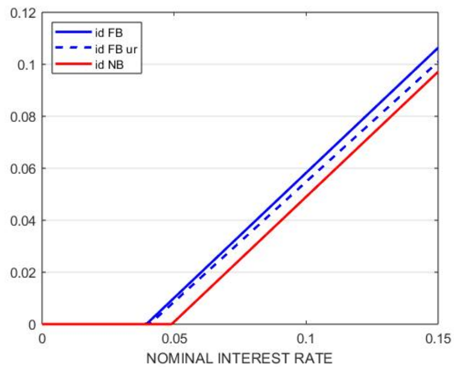
[28]The classic (narrow) corridor corresponds to the average corridor width in the Euro area before (after) the financial crisis in 2008. The corridors imply that central banks pay $i_r < 0$ for low i. This in turn implies that at $i = 0$ $i_d^{PC} < 0$, i.e. for very low i the deposit rate a bank can maximally pay is lower than the return on currency. In this case, banks cannot offer deposits and households switch from holding deposits to holding reserves and no equilibrium with banks exists. The thresholds are $i = 0.0004$ (0.0002) for the classic (narrow) corridor. Although barely visible, the lines for remunerated reserves in figure 6 only start from these thresholds.

Figure 6: Wefare costs of NB: remunerated reserves

(a) Welfare costs



(b) Deposit rates



Notes: Baseline calibration with $i_r = i - s$ where $s = 50(100)$ bps in the narrow (classic) corridor. Deposit rates shown for the classic corridor.

Figure 6a shows that remunerating reserves drastically reduces the welfare costs of 100% reserve NB. For instance, at $i = 15\%$ the welfare costs decrease from 0.6% of GDP with unremunerated reserves to 0.02% (0.03%) of GDP with remunerated reserves in the narrow (classic) corridor. The reason is that remunerating reserves significantly increases deposit rates under NB, as shown in 6b for the classic corridor (for the narrow corridor the increase is even more pronounced). The deposit rates under NB now show a similar behaviour to the FB deposit rates: For low i banks are constrained by the outside option of households, but from around $i = 5\%$ they pay the reserve rate divided by the imperfect competition markup, which proportionally rises with the corridor around i. Deposit rates under FB also increase with $i_r > 0$, but this increase is much smaller than for deposit rates under NB. Under FB, where deposits mainly finance loans, the reserve rate only marginally contributes to the deposit rate, while under NB, where deposits only finance reserves, it is the only source of income. These results confirms the intuition of Friedman (1960), who advocated 100% reserve NB with remunerated reserves already long time ago.

4.3 Effects of an interest-bearing CBDC

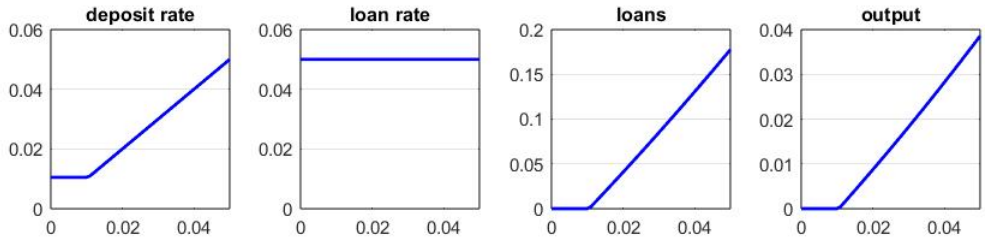
Finally we briefly sketch the quantitative implications when households have access to an interest-bearing CBDC with nominal interest $i_z \geq 0$. With a CBDC, banks equilibrium deposit rate becomes:

$$i_d = \max\left\{ i_z, \frac{1 + i_d^{PC}}{1 + \frac{\sigma}{1-\sigma}\frac{1}{N}} - 1 \right\} \leq i_d^{PC}. \tag{60}$$

The i_z in (60) reflects the fact that the outside option of households to holding deposits is no longer holding unremunerated reserves but holding remunerated CBDC.

Figure 7 shows the effects of increasing the CBDC rate i_z on the deposit rate, the loan rate, real lending and output for a nominal interest rate of 5% and zero interest on reserves.[29] Loans and output are expressed in percentage changes relative to the equilibrium without a CBDC.

Figure 7: Effects of an interest-bearing CBDC



Notes: Baseline calibration with $i = 5\%$ and $i_r = 0$.

First look at the deposit rate in the first graph of figure 7. The flat part is again the imperfect competition deposit rate banks charge without a CBDC, which is around 0.9% at the baseline calibration and $i = 5\%$. For CBDC rates below 0.9%, the CBDC does not affect the economy. If the CBDC rate rises above however, this forces banks to increase the deposit rate one-to-one with the CBDC rate. This is the disciplining role of a CBDC, which counteracts imperfect competition, also highlighted in Chiu et al. (2023).

In contrast to Chiu et al. (2023) or Keister and Sanches (2022) however, this increase in funding costs does not affect the loan rate, i.e. it does not lead to disintermediation. In fact, it even increases or *crowds-in* lending and output as shown in the last two graphs of figure 7. In Chiu et al. (2023) this only happens for low CBDC rates, while higher CBDC rates cause disintermediation. What prevents disintermediation here, as already highlighted in section 3.4, is that banks have economically useful financing alternatives to deposits. This mechanism is either absent or not essential in the other models. In Chiu et al. (2023) financing lending over bonds – although it is possible – is not used in equilibrium and in Keister and Sanches (2022) banks cannot issue other liabilities than deposits. Here, with higher deposit rates, holding deposits becomes more attractive, banks can lend out these additional deposits, lending and output increase and so does welfare, again because households are better compensated against inflation.

[29] Real lending is given by (25) and output is simply the sum of FM and SM output, $(1-s)q_b+B$. The quantitative effects with remunerated reserves are very similar.

5 Conclusion

The paper studies what is at stake if we move from the current FB system, with its characteristic combination of financing investment or lending with monetary liabilities like deposits, to 100% reserve NB or a system with a CBDC, which constrain banks' money creation. The paper argues that in contrast to widespread concerns e.g. in CBDC discussions, if banks have economically viable financing alternatives to deposits, constraining FB must not lead to bank disintermediaion, i.e. a crowding-out of investment and lending. Instead, the welfare costs of constraining FB mainly arise from lower interest on deposits, in particular if the banking sector is competitive and inflation is high. FB is then able to better compensate deposit-holders against inflation. The calibration suggests however, that these benefits are only sizeable for high inflation rates and they drastically decrease if the central bank pays interest on reserves. To the extent that NB regulations mitigate the instability issues associated with the financing structure of FB, increasing financial stability with a NB system might thus not be so costly.

References

Aleksander Berentsen, Gabriele Camera, and Christopher Waller. Money, credit and banking. *Journal of Economic Theory*, 135(1):171 – 195, 2007.

Aleksander Berentsen, Samuel Huber, and Alessandro Marchesiani. Financial Innovations, Money Demand, and the Welfare Cost of Inflation. *Journal of Money, Credit and Banking*, 47(S2): 223–261, June 2015.

Markus K. Brunnermeier and Dirk Niepelt. On the equivalence of private and public money. *Journal of Monetary Economics*, 106:27 – 41, 2019.

Stephen Ceccetti and Kermit Schoenholtz. Narrow banks wont stop bank runs. `https://www.moneyandbanking.com/commentary/2014/4/28/narrow-banks-wont-stop-bank-runs`, 04 2014.

V.V. Chari and Christopher Phelan. On the social usefulness of fractional reserve banking. *Journal of Monetary Economics*, 65:1 – 13, 2014.

Jonathan Chiu, Seyed Mohammadreza Davoodalhosseini, Janet Jiang, and Yu Zhu. Bank market power and central bank digital currency: Theory and quantitative assessment. *Journal of Political Economy*, 131(5):1213–1248, 2023.

Ben Craig and Guillaume Rocheteau. Inflation and Welfare: A Search Approach. *Journal of Money, Credit and Banking*, 40(1):89–119, February 2008.

Douglas W. Diamond and Philip H. Dybvig. Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy*, 91(3):401 – 419, 1983.

Xavier Freixas and Jean-Charles Rochet. *Microeconomics of Banking, 2nd Edition*, volume 1 of *MIT Press Books*. The MIT Press, 2 edition, February 2008.

Milton Friedman. *A Program for Monetary Stability*. Fordham University Press, 1960.

Alberto Di Iorio, Anneke Kosse, and Ilaria Mattei. *Embracing diversity, advancing together - results of the 2023 BIS survey on central bank digital currencies and crypto*. Number 147 in BIS Papers. Bank for International Settlements, 2024.

Timothy Jackson and George Pennacchi. How should governments create liquidity? *Journal of Monetary Economics*, 118(C):281–295, 2021.

Todd Keister and Daniel Sanches. Should Central Banks Issue Digital Currency? *The Review of Economic Studies*, 90(1):404–431, 03 2022. ISSN 0034-6527.

Ricardo Lagos and Randall Wright. A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3):463–484, 2005.

Jr. Lucas, Robert E. Glass-steagall: A requiem. *American Economic Review*, 103(3):43–47, May 2013.

Robert E. Lucas and Juan Pablo Nicolini. On the stability of money demand. *Journal of Monetary Economics*, 73(C):48–65, 2015.

Robert E. Lucas, Jr. Inflation and welfare. *Econometrica*, 68(2):247–274, 2000.

Cyril Monnet, Hyun Song Shin, Jon Frost, Leonardo Gambacorta, Raphael Auer, and Tara Rice. Central Bank Digital Currencies: Motives, Economic Implications, and the Research Frontier. *Annual Review of Economics*, 14(1):697–721, August 2022.

George Pennacchi. Narrow banking. *Annual Review of Financial Economics*, 4(1):141–159, 2012.

Guillaume Rocheteau and Ed Nosal. *Money, Payments, and Liquidity*. MIT Press, 2017.

Toni M. Whited, Yufeng Wu, and Kairong Xiao. Will Central Bank Digital Currency Disintermediate Banks? Working paper, 2022.

Stephen Williamson. Central bank digital currency: Welfare and policy implications. *Journal of Political Economy*, 130(11):2829–2861, 2022.

Appendix A

A.1 Perfect competition economy without banks and preference shock

In this economy there are no banks and there is a fixed measure s of sellers and $1-s$ of buyers, i.e. there is no preference shock. Money is provided by the central bank in the form of reserves at rate i_r. This is a basic version of Rocheteau and Nosal (2017) with interest on central bank money. In such an environment only buyers acquire reserves if holding money is costly. Their optimal reserve holdings read, analogue to (35):

$$\phi = \beta V_b'(m_{+1}) = \beta \frac{u'(q_{b+1})(1+i_{r+1})}{p_{+1}}$$

Using (29) and stationarity this becomes:

$$\frac{u'(q_b)}{c'(\frac{1-s}{s}q_b)} = \frac{1+i}{1+i_r}. \tag{61}$$

This is the same allocation as in the perfect competition model with banks and preference shocks under NB, i.e. as (43) with $\alpha = 1$.

A.2 Perfect competition economy without bonds

Suppose the environment is identical to section 3, but banks can only issue deposits as liabilities. The main implication is that sellers cannot hold bonds now and thus also hold deposits. Their marginal value of holding reserves, (34), thus becomes $V_s'(m) = \phi(1+i_d)$ and the optimality condition for reserve holdings, (35), changes to

$$\phi = \beta s \phi_{+1}(1+i_{d+1}) + \beta(1-s)\phi_{+1}(1+i_{l+1}),$$

which becomes under stationarity:

$$i = si_d + (1-s)i_l \tag{62}$$

Since the deposit-pricing condition for banks, (6), still holds here, the equilibrium loan rate i_l increases in α as follows:

$$i_l = \frac{i - s\alpha i_r}{1 - s\alpha} \tag{63}$$

Note that this additional spread between loan and deposit rate also translates into lower consumption in (43) and welfare.

Appendix B

B.1 proof of proposition 2

Proof. Apart from the determination of the deposit rate, (51), the equilibrium conditions under imperfect competition are the same as under perfect competition. In particular, $i_l = i_b = i$ still holds, and q_b solves (43). This implies total real deposits, ϕD are independent of N and $i_d^{PC} = \alpha i_r + (1-\alpha)i$, thus $i_{d,FB}^{PC} > i_{d,NB}^{PC} = i_r$.

To proof proposition 2 note from (50) that if N increases from $\bar{N}$ the right-hand side of (50) decreases. Thus (50) can only hold if the left-hand side decreases too which implies i_d^* must increase or $\partial i_d^*/\partial N > 0$. Therefore $i_d^* > 0$ if $N > \bar{N}$. Conversely for $N < \bar{N}$, $i_d^* < 0$ and thus $i_d = 0$.

Now apply this to *ii*), when $N \leq \bar{N}_{FB}$. Note that $i_{d,FB}^{PC} > i_{d,NB}^{PC}$ implies $\bar{N}_{FB} < \bar{N}_{NB}$ since $\bar{N}$ decreases in i_d^{PC} from (52). Thus if $N \leq \bar{N}_{FB}$ $N < \bar{N}_{NB}$ also holds, which implies $i_{d,NB}^* < 0$ and $i_{d,FB}^* < 0$ and thus $i_{d,FB} = i_{d,NB} = 0$.

If $N > \bar{N}_{FB}$ we have two cases: Either $\bar{N}_{FB} < N < \bar{N}_{NB}$ or $N > \bar{N}_{NB}$. The first case implies $i_{d,NB}^* < 0$ and thus $i_{d,NB} = 0$ and $i_{d,FB} = i_{d,FB}^* > 0$. Thus $i_{d,FB} > i_{d,NB}$ holds.

In the second case both $i_{d,FB} = i_{d,FB}^* > 0$ and $i_{d,NB} = i_{d,NB}^* > 0$. To show $i_{d,FB}^* > i_{d,NB}^*$ reformulate (50) as

$$i_d^{PC} = i_d + \frac{i_d}{\varepsilon_D(i_d)}\frac{1}{N} = i_d + \frac{\phi D}{(\partial \phi D/\partial i_d)}\frac{1}{N} = i_d + m(i_d) = RHS(i_d), \tag{64}$$

where $m(i_d)$ denotes the markup between the perfect and the imperfect competition rate, and $RHS(i_d)$ is the right-hand side of (64). From (48) ϕD and $\partial \phi D/\partial i_d$ read

$$\phi D = (1-s)\frac{q_b u'(q_b)}{1+i} \tag{65}$$

$$\frac{\partial \phi D}{\partial i_d} = (1-s)\frac{(\partial q_b/\partial i_d)\left(u'(q_b) + q_b u''(q_b)\right)}{1+i} > 0, \tag{66}$$

where the positive derivative follows from the assumption that the coefficient of relative risk aversion of buyers is below one, i.e. $\sigma = -\frac{q_b u''(q_b)}{u'(q_b)} < 1$. Thus we can rewrite $m(i_d)$ as:

$$m(i_d) = \frac{\phi D}{(\partial \phi D/\partial i_d)}\frac{1}{N} = \frac{q_b}{(\partial q/\partial i_d)}\frac{u'(q_b)}{(u'(q_b) + q_b u''(q_b))}\frac{1}{N} = \frac{q_b}{(\partial q_b/\partial i_d)}\frac{1}{(1-\sigma)}\frac{1}{N}. \tag{67}$$

Now differentiate both sides of (43) with respect to i_d to get:

$$\frac{\partial q_b}{\partial i_d} = -\frac{1+i}{(1+i_d)^2}\frac{1}{u''(q_b)} = \frac{q_b}{(1+i_d)}\frac{1}{\sigma}. \tag{68}$$

Thus (67) becomes

$$m(i_d) = \frac{q_b}{(\partial q_b/\partial i_d)}\frac{1}{(1-\sigma)}\frac{1}{N} = \frac{\sigma}{(1-\sigma)}(1+i_d) \tag{69}$$

and $RHS(i_d)$ linearly increases in i_d. Ergo $RHS(i^*_{d,FB}) = i^{PC}_{d,FB} > RHS(i^*_{d,NB}) = i^{PC}_{d,NB}$ implies $i^*_{d,FB} > i^*_{d,NB}$.

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